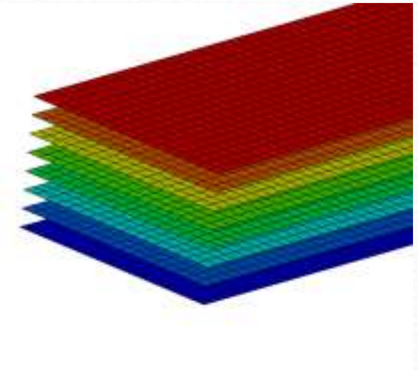
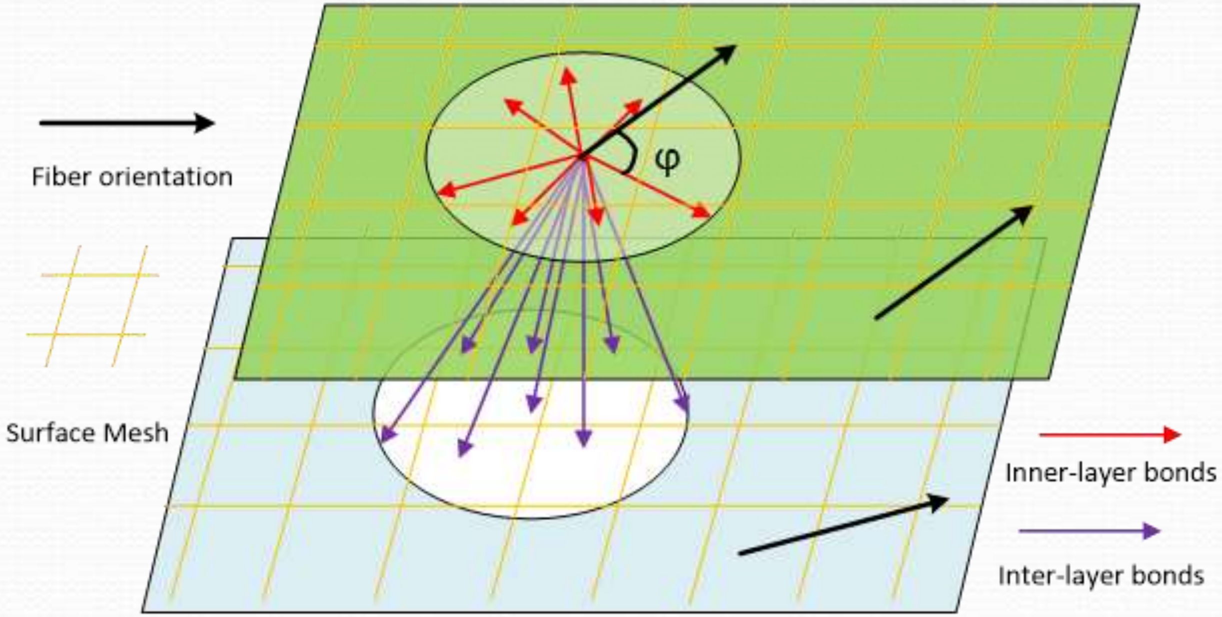
3. The Peridynamic Laminate Model (1)

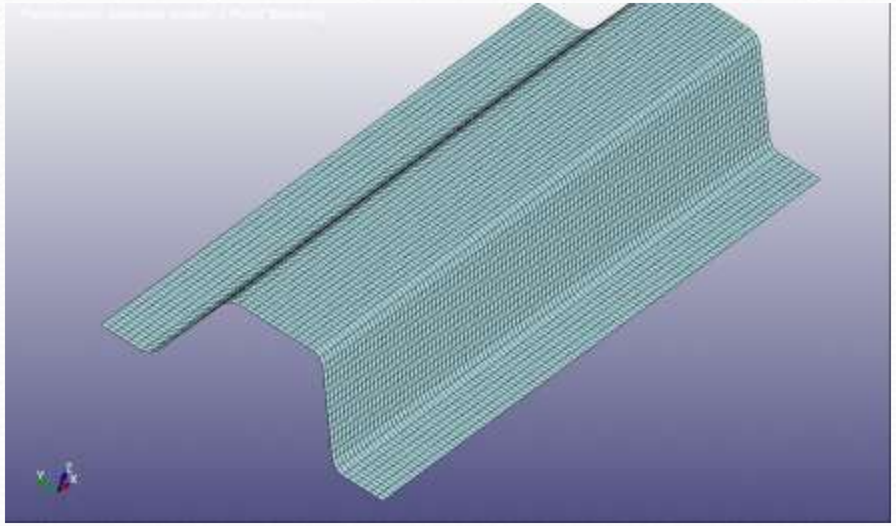
1) Computational mesh: a sequence of surface mesh.
2) Each lamina is a plane stress structure with a transversely isotropic material.
3) A material point in a lamina can only interact with points inside that lamina and its adjacent laminas
4) Material: material property $c(\varphi)$ is a distributive function.

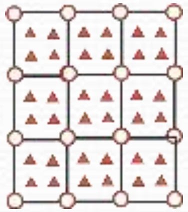




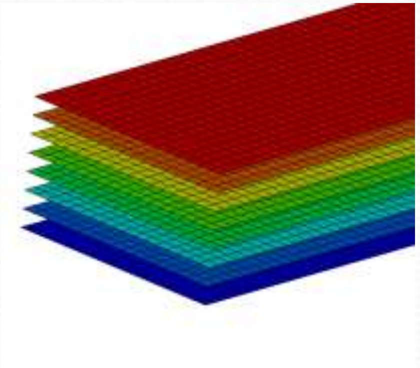
3. The Peridynamic Laminate Model (2)

1) Computational mesh: a sequence of surface mesh.



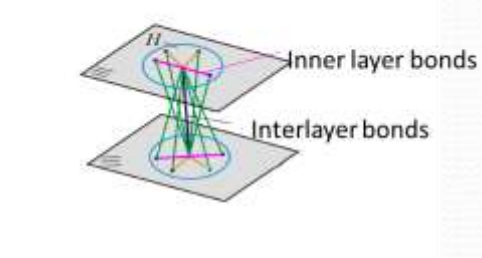


2) Gaussian integration is adopted.
The volumes of Gaussian point is determinated by surface area and ply thickness



3) Time step is controlled by the dimension of surface element.

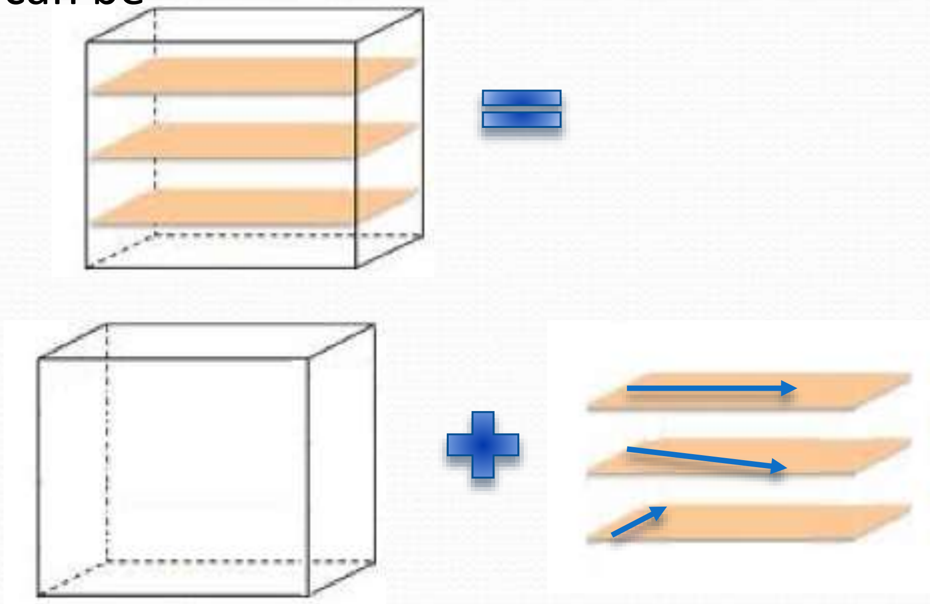
4. The Micro Modulus (1)



The elastic matrix of laminate material can be **decomposed** to two parts:

$$\boldsymbol{D} = \boldsymbol{D}^{mt} + \boldsymbol{D}^{ft}$$

$$c(\varphi) = c_{mt} + c_{fb}(\varphi)$$



$\boldsymbol{D}^{mt}$: Isotropic material.
Determine micro-elastic modulus c_{mt} for all bonds

$\boldsymbol{D}^{ft}$: transverse isotropic.
Determine micro-elastic modulus $c_{fb}(\varphi)$ for inner layer bonds, which has **zero** stiffness along the vertical direction of the fiber

$$\boldsymbol{D}^m = \begin{bmatrix} \frac{E_p}{1-v^2} & \frac{vE_p}{1-v^2} & 0 \\ \frac{vE_p}{1-v^2} & \frac{E_p}{1-v^2} & 0 \\ 0 & 0 & \frac{E_p}{2(1+v)} \end{bmatrix}$$

$$\boldsymbol{D}^f = \boldsymbol{D}_{2D} - \boldsymbol{D}^m$$

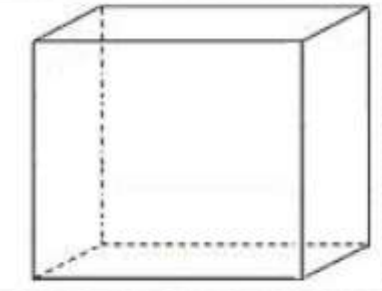
$$v = v_{tp}\sqrt{E_p/E_t}$$

4. The Micro Modulus (2)

The engineering material constants of a laminate are: E_f, E_m, v_{fm}, G_{fm}.
The isotropic matrix material ($\boldsymbol{D}^{mt}$) is constructed by E_m and v_{fm}

The elastic energy density from classic mechanics theory with matrix material is:

$$w^{clmt} = \frac{1}{2} D_{ij}^{mt} \epsilon_j \epsilon_i$$



$\boldsymbol{D}^{mt}$

The elastic energy density from peridynamic theory is:

$$w^{pdmt} = \frac{1}{2} \int \frac{1}{2} c^{mt} s^2 \xi \text{ dv+dv}$$

$$w^{clmt} = w^{pdmt} \quad \Rightarrow \quad c^{mt}$$

4. The Micro Modulus (3)

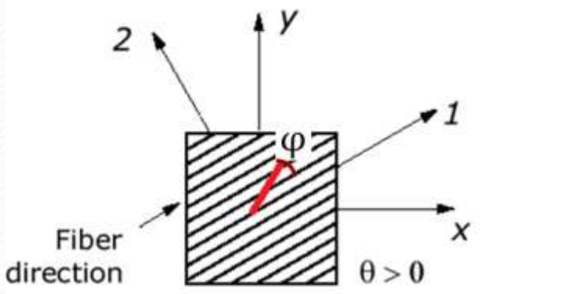
$$\boldsymbol{D}^{f} = \boldsymbol{D} - \boldsymbol{D}^{mt}$$

$\boldsymbol{D}$ is the total material matrix for each lamina which is constructed by E_f, E_m, v_{fm}, G_{fm}

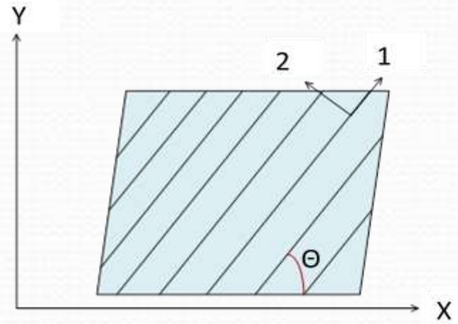
$\boldsymbol{D}^{f}$ is a transversely isotropic material with $c(90)$=0

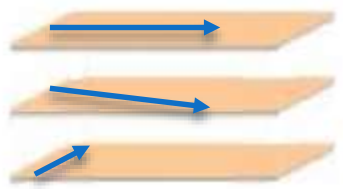
$$\boldsymbol{D}^{ft} = \boldsymbol{T}^{-1} \cdot \boldsymbol{D}^{f} \cdot \boldsymbol{R} \cdot \boldsymbol{T},$$

$$\boldsymbol{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \text{ and } \boldsymbol{T} = \begin{bmatrix} C^2 & S^2 & 2CS \\ S^2 & C^2 & -2CS \\ -CS & CS & C^2 - S^2 \end{bmatrix}$$



Transverse isotropic
With $c(90)$=0

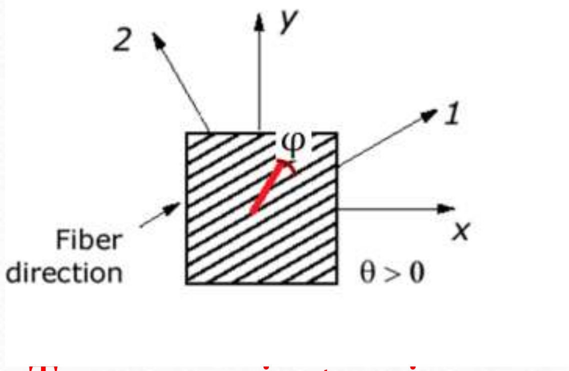




4. The Micro Modulus (4)

The elastic energy density from classic mechanics theory with $\boldsymbol{D}^{fb}$ is:

$$w^{clfb} = \frac{1}{2} D_{ij}^{ft} \epsilon_j \epsilon_i$$



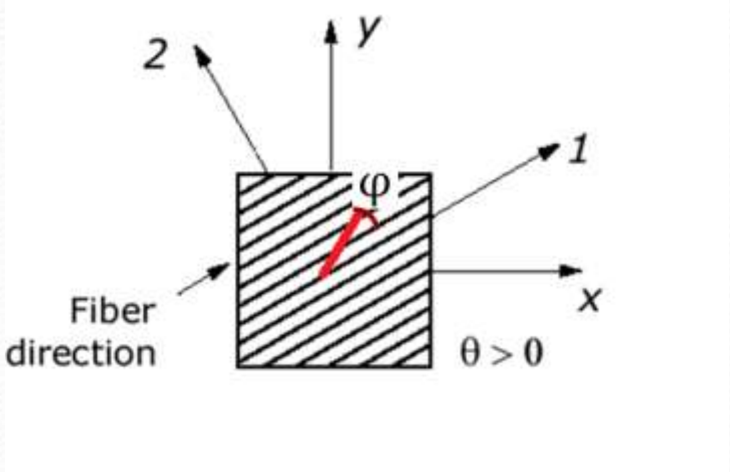
Transverse isotropic
With $c(90)$=0

The elastic energy density from peridynamic theory with c^{fb} is:

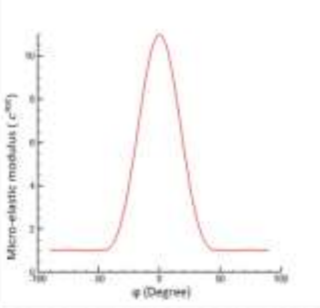
$$w^{pdfb} = \frac{1}{2} \int \frac{1}{2} c^{fb}(\varphi) s^2 \xi \, \mathrm{dv}$$

4. The Micro Modulus (5)

The micro elastic modulus of a inner layer bond ($c^{fb}(\varphi)$) is a distributive function of the angle between bond and fiber. Expanded by the **8th order spherical harmonic expansion** (Ghajari, etc, 2014):



$$c^{fb}(\varphi) = A_{00} + A_{20}P_2^0(cos\varphi) + A_{40}P_4^0(cos\varphi) + A_{60}P_6^0(cos\varphi) + A_{80}P_8^0(cos\varphi)$$



$$c^{fb}(0) = c_1 \quad c^{fb}(45) = 0 \; c^{fb}(55) = 0 \quad c^{fb}(65) = 0 \; c^{fb}(90) = 00$$

$$A_{00} = 0.0722c_1$$
$$A_{20} = 0.3021c_1$$
$$A_{40} = 0.3376c_1$$
$$A_{60} = 0.2159c_1$$
$$A_{80} = 0.0722c_1$$

$$P_2^0(cos\varphi) = \frac{1}{2}(3cos^2\varphi - 1)$$

$$P_4^0(cos\varphi) = \frac{1}{8}(35cos^4\varphi - 30cos^2\varphi + 3)$$

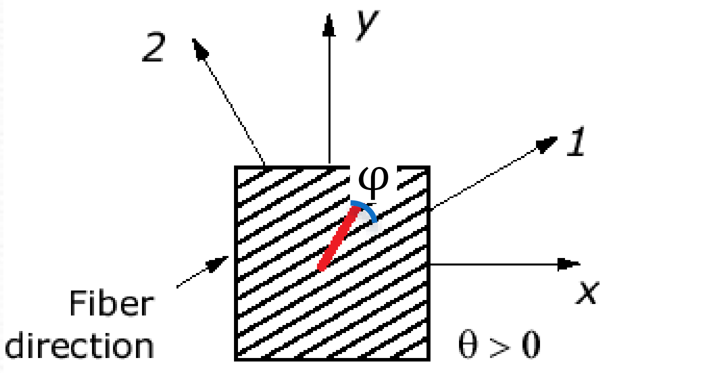
$$P_6^0(cos\varphi) = \frac{1}{16}(231cos^6\varphi - 315cos^4\varphi + 105cos^2\varphi - 5)$$

$$P_8^0(cos\varphi) = \frac{1}{128}(6435cos^8\varphi - 12012cos^6\varphi + 6930cos^4\varphi - 1260cos^2\varphi + 35)$$

4. The Micro Modulus (6)

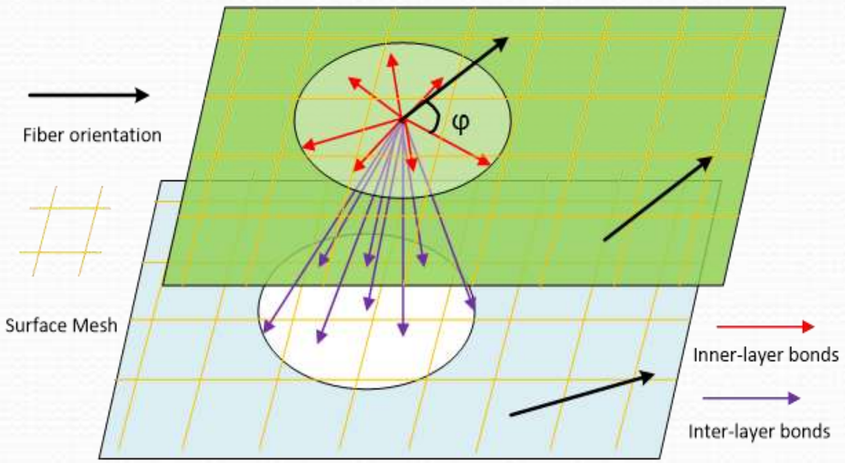
By the equivalent elastic energy density:

$w^{clfb} = w^{pdfb}$ ⇒ $c^{fb}(\varphi)$



The micro-modulus of inter-layer bonds: $c = c^{mt}$

The micro-modulus of inner-layer bonds: $c(\varphi) = c^{mt} + c^{fb}(\varphi)$



5. The Critical Bond Stretch (1)

For failure analysis, the critical bond stretch is also a distributive function of fiber angle $s(\varphi)$ which as be expanded with a harmonic expansion form:

$$s_0^2(\varphi) = B_{00} + B_{20}P_2^0(cos\varphi) + B_{40}P_4^0(cos\varphi) + B_{60}P_6^0(cos\varphi) + B_{80}P_8^0(cos\varphi)$$

$$B_{00} = 0.0722s_{01}^2 + 0.9278s_{02}^2$$

$$B_{20} = 0.3021(s_{01}^2 - s_{02}^2)$$

$$B_{40} = 0.3376(s_{01}^2 - s_{02}^2)$$

$$B_{60} = 0.2159(s_{01}^2 - s_{02}^2)$$

$$B_{80} = 0.0722(s_{01}^2 - s_{02}^2)$$

5. The Critical Bond Stretch (2)

The energy release rate can related with s_0^2:

$$G_{Ic1} = \int_0^{\delta}\int_z^{\delta}\int_{-c0s^{-1}(\frac{z}{\xi})}^{c0s^{-1}(\frac{z}{\xi})}\left[\frac{c(\varphi)s_0^2(\varphi)\xi}{2}\right]t\xi d\varphi d\xi dz$$

$$G_{Ic2} = \int_0^{\delta}\int_z^{\delta}\int_{sin^{-1}(\frac{z}{\xi})}^{\pi-sin^{-1}(\frac{z}{\xi})}\left[\frac{c(\varphi)s_0^2(\varphi)\xi}{2}\right]t\xi d\varphi d\xi dz$$

And it leads:

$$s_{01}^2 = \frac{500[(4G_{Ic1} - 11G_{Ic2})c_1 + (112G_{Ic1} - 72G_{Ic2})c_2]}{t\xi^4(71c_1^2 + 3168c_1c_2 + 944c_2^2)}$$

$$s_{02}^2 = \frac{500[(31.5G_{Ic2} - 5G_{Ic1})c_1 + (11G_{Ic2} - 4G_{Ic1})c_2]}{t\xi^4(71c_1^2 + 3168c_1c_2 + 944c_2^2)}$$