Approximation of crack in element

$$\mathbf{u}^h(\mathbf{X},t) = \sum_I N_I(\mathbf{X})\left\{\mathbf{u}_I(t) + \mathbf{q}_I(t)\left[H(f(\mathbf{X})) - H(f(\mathbf{X}_I))\right]\right\}$$
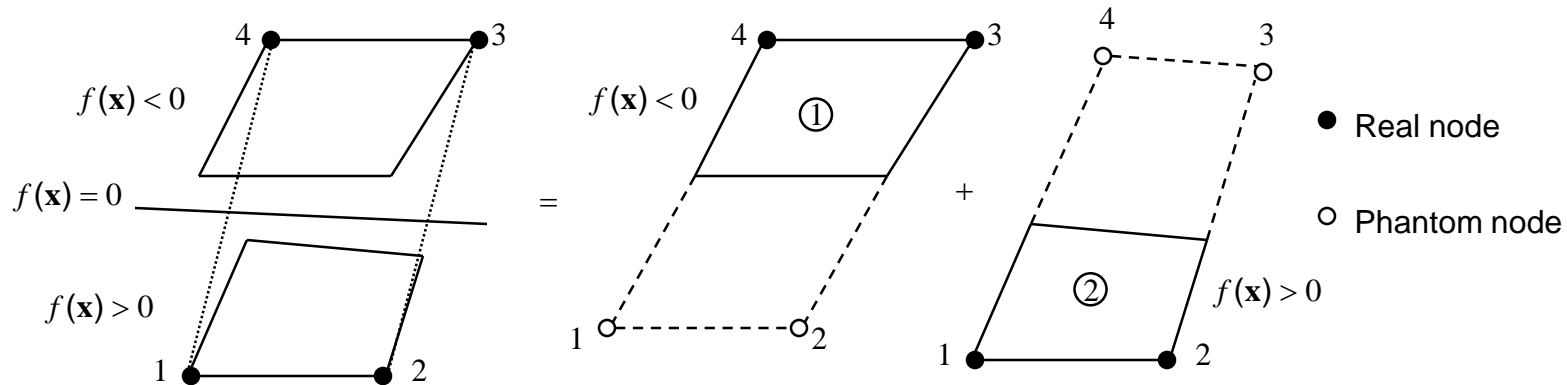
Rewrite into superposition of two phantom elements

$$\mathbf{u}^h(\mathbf{X},t) = \underbrace{\sum_{I \in S_1} \mathbf{u}_I^1(t) N_4(\mathbf{X}) H(-f(\mathbf{X}))}_{\mathbf{u}^1(\mathbf{X},t)} + \underbrace{\sum_{I \in S_2} \mathbf{u}_I^2(t) N_4(\mathbf{X}) H(f(\mathbf{X}))}_{\mathbf{u}^2(\mathbf{X},t)}$$

where

$$\mathbf{u}_I^1 = \begin{cases} \mathbf{u}_I & \text{if } f(\mathbf{X}_I) < 0 \\ \mathbf{u}_I - \mathbf{q}_I & \text{if } f(\mathbf{X}_I) > 0 \end{cases}$$

$$\mathbf{u}_I^2 = \begin{cases} \mathbf{u}_I + \mathbf{q}_I & \text{if } f(\mathbf{X}_I) < 0 \\ \mathbf{u}_I & \text{if } f(\mathbf{X}_I) > 0 \end{cases}$$

Phantom Element Integration

- Song, Areias and Belytschko (2006)

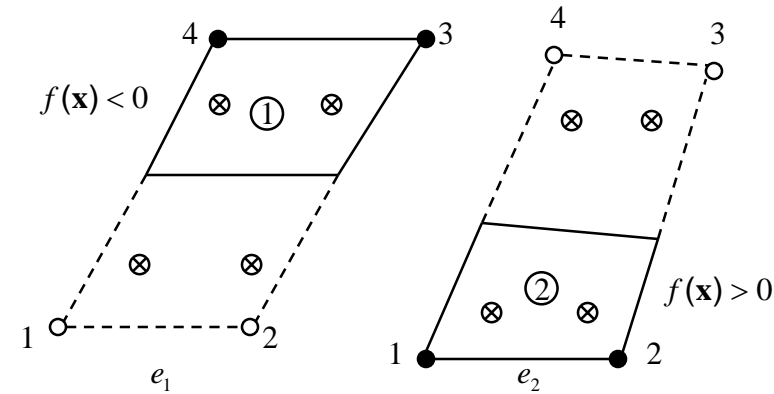Integrate in phantom elements and assemble according to area ratios

$$\mathbf{f}^{kin}_{(e_1/e_2)} = \frac{A_{(e_1/e_2)}}{A_0} \int_{\Omega^e_0} \rho_0 \mathbf{N}^T \mathbf{N} d\Omega^e_0 \ddot{\mathbf{x}}_{(e_1/e_2)}$$

$$\mathbf{f}^{int}_{(e_1/e_2)} = \frac{A_{(e_1/e_2)}}{A_0} \int_{\Omega^e_0} \mathbf{B}^T \mathbf{P} d\Omega^e_0$$

$$\mathbf{f}^{ext}_{e_1} = \frac{A_{e_1}}{A_0} \int_{\Omega^e_0} \rho_0 \mathbf{N}^T \mathbf{b} d\Omega^e_0 + \int_{\Gamma^e_{0,t}} \mathbf{N}^T \mathbf{t}^0 H[-f(\mathbf{X})] d\Gamma^e_{0,t}$$

$$\mathbf{f}^{ext}_{e_2} = \frac{A_{e_2}}{A_0} \int_{\Omega^e_0} \rho_0 \mathbf{N}^T \mathbf{b} d\Omega^e_0 + \int_{\Gamma^e_{0,t}} \mathbf{N}^T \mathbf{t}^0 H[f(\mathbf{X})] d\Gamma^e_{0,t}$$

$$\mathbf{f}^{coh}_{e_1} = -\int_{\Gamma^e_{0,c}} \mathbf{N}^T \boldsymbol{\tau}^c \mathbf{n}_0 d\Gamma^e_{0,c} \qquad \mathbf{f}^{coh}_{e_2} = \int_{\Gamma^e_{0,c}} \mathbf{N}^T \boldsymbol{\tau}^c \mathbf{n}_0 d\Gamma^e_{0,c}$$

Easier in implementation

Sub-domain integration

Integration conducted in two sub-domains cut by cracks

More accurate results; available in 2D XFEM

Difficulties in implementation: Varied integration schemes,
Different data structure, Transfer of state variables