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Highlights

- Three dimensional smooth particle Galerkin formulation is presented
- An adaptive anisotropic Lagrangian kernel is utilized
- A bond-based failure criterion is introduced
- A frictionless self-contact algorithm has been developed
- Numerical results are validated with test data for high velocity impact penetration on concrete structures

Three-dimensional Concrete Impact and Penetration Simulations Using the Smoothed Particle Galerkin Method

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Abstract

In this paper, we model the three-dimensional concrete impact and penetration problems using a stabilized meshfree method. The present method is established using a non-residual penalty term from strain smoothing as a means of stabilizing the meshfree nodal integration method under the Galerkin framework. As a result, the meshfree discretization leads to a dual stress point algorithm with the stabilization parameterized by a measure of a local length scale. An adaptive anisotropic Lagrangian kernel is considered in junction with the stabilized meshfree formulation for the severe deformation analysis. In order to avoid the spurious damage growth and material self-healing in concrete failure analysis, a bond-based failure criterion is introduced. A frictionless self-contact algorithm is also developed to model the interaction between concrete debris in damage. Several impact examples are investigated including the study of scabbing and perforation of concrete under high velocity impact. The numerical results are compared with the experimental data to demonstrate the effectiveness of the present method.

Keywords: Smoothed particle Galerkin; Impact penetration; Concrete; Meshfree

1. Introduction

The necessity to model the severe deformation followed by moving discontinuities in three-dimensional concrete impact and penetration problems makes numerical simulation extremely difficult by the conventional mesh-based numerical methods such as the Lagrangian finite element method. Although the Eulerian description for fluid mechanics applications can be easily adopted to circumvent the mesh distortion problem encountered in the Lagrangian formulation, the Eulerian representation of material flow presents other numerical difficulties in tracking the material points and free surfaces in concrete impact and penetration simulations. Alternatively, the arbitrary Lagrangian Eulerian (ALE) algorithm [1] advances the mesh independently with material flow and makes it possible to take into account the movements of free surfaces while reducing mesh distortion. Regardless of this distinct property offered by ALE method, its shortcoming is the occurrence of numerical oscillations when the convective effect is dominant

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in the governing equations. This numerical instability often arises in impact and penetration simulation when the velocity difference between the mesh movements and medium flow becomes evident. As a generalization of Eulerian approach, the ALE method also has difficulty to model the formation of new surfaces in the course of perforation and fragmentation processes.

Meshfree methods, on the other hand, offer diverse numerical advantages over the conventional mesh-based numerical methods in modeling large deformation [2, 3], moving discontinuity [4] and immersed problems [5]. Meshfree methods can be roughly categorized into discontinuous and continuous approaches. The Discrete Element Method (DEM) [6] is a representative discontinuous meshfree method based on the contact interactions between discrete grains to model the motion of granular materials. In contrast to DEM that describes the problem at particle scale, the majority of meshfree methods belong to the continuous approach that discretizes the problem at a field-scale level [7]. In essence, the material behavior in continuous meshfree methods is endowed with the conservation properties (mass, momentum and energy), and the uniqueness and convergence of solutions are ensured under appropriate variational principle and relevant field approximations. Especially, an accurate constitutive model for continuous meshfree methods is desired to obtain sufficient information of stress and deformation fields for capturing the fundamental structure response as well as the projectile characteristics in concrete impact and penetration simulations.

The earliest development in the continuous meshfree methods is the Smoothed Particle Hydrodynamics (SPH) method. The foundation of the SPH method is the kernel estimate introduced by Monaghan [8]. In SPH method, partial differential equations are transformed into integral equations and the kernel estimate then provides the approximation to estimate the field variables at the discrete particles. Since the functions are evaluated at the particles, the use of a mesh is no longer required. This ability to handle severe deformations without the use of meshes in fluid-like motion allows the SPH method to be applied to problems that historically have been reserved for Eulerian approaches [9]. In spite of that, a direct application of SPH method to solid mechanics problems is known to suffer numerous numerical deficiencies [10, 11], namely the lack of approximation consistency, tension instability, presence of spurious low-energy modes, dispersive wave propagation, complication in enforcing the essential boundary conditions, difficulty to represent the crack surface in 3D problem and inability to prevent the material self-healing in failure analysis. While many SPH models [12 -14] have been utilized to simulate the severe deformation and material failure in concrete impact and penetration problems, less attention [7, 15, 16] was paid to the improvement of the unstable results induced by those numerical deficiencies.

In the past two decades, many advanced meshfree Galerkin methods were developed to resolve some of the numerical issues in SPH method. Among them, Element-free Galerkin (EFG) [17] and Reproducing Kernel Particle Method (RKPM) [18] are two earliest meshfree methods to tackle the issues of approximation consistency and boundary conditions in SPH method. In addition to the improvement of approximation consistency, a Galerkin-based SPH formulation [19] was developed to intensify the formulation consistency for the analysis of free surface flows. In the meantime, the Cracking Particles Method (CPM) was developed [20] to provide a simple way for the description of evolving cracks in brittle fracture analysis. As an alternative to the visibility method [1] and the methods based on geometric information or screening effects [22], a discontinuous representation of the crack surface in CPM is delineated through the enrichment of step function at cracking particles. This unique feature of CPM makes it attractive to model the complex fracture behavior in three-dimensional case. The Stabilized

Conforming Nodal Integration (SCNI) method [23] is another representative meshfree method introduced to ameliorate the spurious low-energy modes present in meshfree direct nodal integration scheme. The SCNI technique was later generalized to higher order strain smoothing by Duan *et al.* [24] to achieve cubic rates of convergence in the L^2 norm for the linear analysis. Arbitrary high order Galerkin exactness for SCNI method was also derived [25] under the general framework of variational consistency for improving the integration errors in meshfree methods. In the meantime, different integration schemes [26, 27] based on SCNI method also have been proposed to enhance the numerical stability of SCNI method. A modification of SCNI algorithm to the stabilized non-conforming nodal integration (SNNI) scheme [28] also has been developed to bypass the need of constructing the conforming smoothing cells for nodal integration. Despite the irreconcilable demands on the stabilization control parameter placed by the accuracy requirement, the SNNI scheme has been successfully applied to the model of the concrete impact and penetration problems [29]. Several regularized meshfree methods [30 – 32] were also developed based on the concept of SCNI method for the material failure analysis. Nevertheless, most meshfree methods cannot preclude the use of background mesh or smoothing cells, thus pose significant challenges from both the mathematical formulation and the programming aspects in the three-dimensional simulation of severe deformation and material failure problems.

The Smoothed Particle Galerkin (SPG) method [33, 34] recently introduced by Wu *et al*. is a new continuous meshfree method that aims to bypass the need of background mesh, reveal a proper stabilization setting and guide the development of practical nodal integration schemes for severe deformation problems. The essence of SPG method is an introduction of strain operator for stabilization. Most notably, the strain operator is invertible stable and well-defined in the severe deformation analysis [34]. In the earliest SPG method [33], the strain operator is defined through a strain gradient stabilization (SGS) scheme [33, 34]. In SGS scheme, the first-order strain gradients are derived based on the decomposed strain field from the displacement smoothing. In the subsequent SPG method [35, 36], the first-order strain gradients are derived based on a direct strain smoothing leading to a penalty-based h^2-stabilization formulation. As opposed to the residual type stabilization [28, 37], the SPG stabilization formulation is a non-residual type in which the penalized stabilization functional is parameterized by a measure of the local length scale without a need of stabilization control parameter. Following that, an incorporation of second-order strain gradients in the SPG formulation has led to a development of the regularized SPG method [35] for the analysis of damage-induced strain localization problem in elastic materials. Most recently, a particle insertion-deletion scheme [36] has been introduced to the regularized SPG method to substantially model the ductile fracture in two-dimensional explicit dynamics analysis.

The scope of this paper is to present an improved SPG method for the prediction of fundamental structure response and projectile characteristics in the three-dimensional concrete impact and penetration problems. The present method combines our previous development in two-dimensional large deformation analysis [34] and several numerical enhancements in failure analysis. The outline of the paper is organized as follows: An overview of smoothed particle Galerkin method for large deformation analysis is given in Section 2. The corresponding three-dimensional discrete equations and resulting dual stress point algorithm for spatial integration are described in Section 3. The adoption of adaptive anisotropic Lagrangian kernel in the three-dimensional formulation is also described in the same section. Section 4 presents a regularized concrete damage model. The consideration of bond-based failure and self-contact between

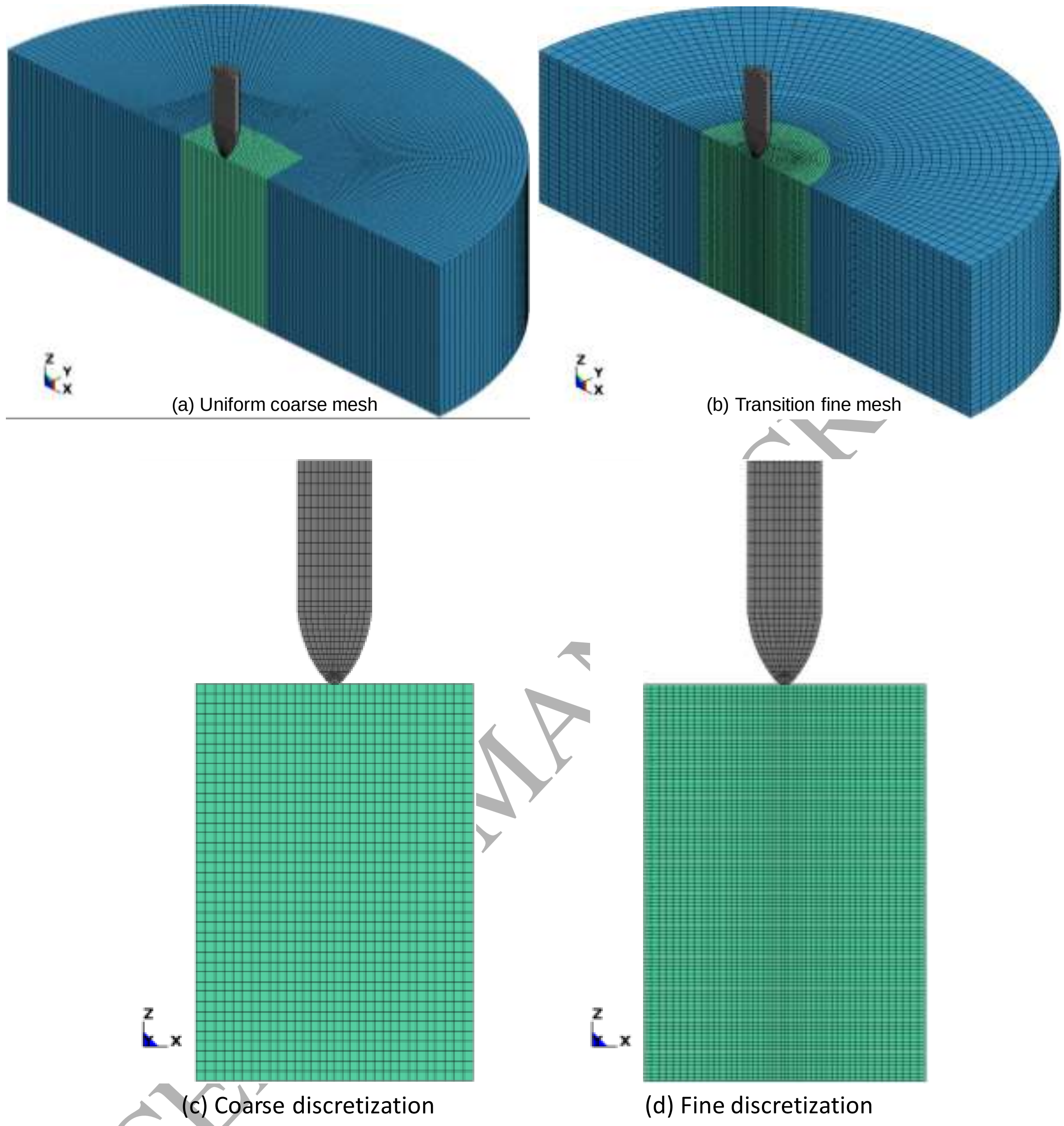

Fig. 13. Perforation responses: refined discretization

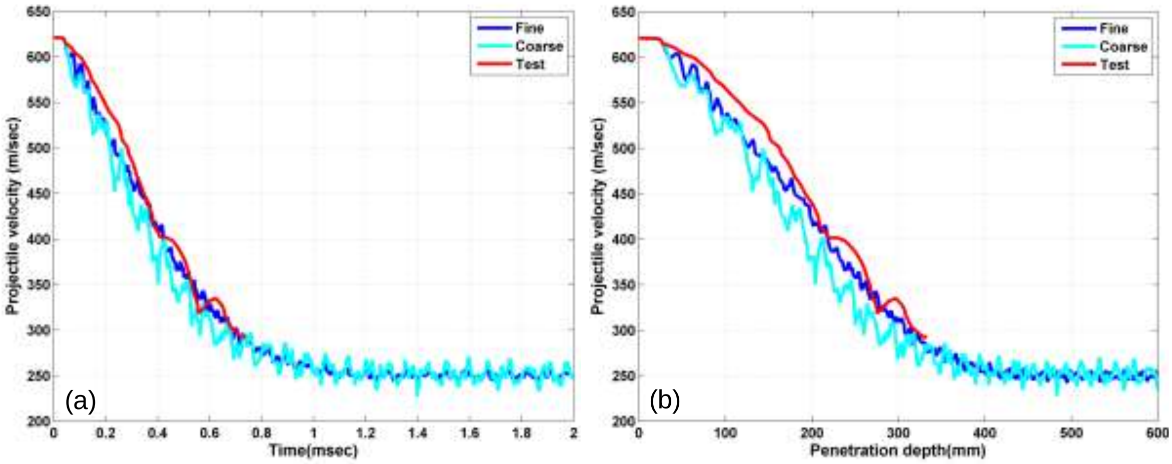

Fig. 14. Perforation responses with refined discretization: (a) projectile velocity time history, (b) history of projectile velocity vs penetration depth.

5.2. Penetration Analysis

In the penetration test, the specimen has a diameter of 1400 mm and length of 800 mm. The projectile was fired at a velocity of 623 m/sec towards the center of the specimen. The geometry and discretization of the model is shown in Fig. 15. Similar to the perforation test, only a small region of the concrete target right under the projectile is discretized by the present method and the rest of area is modeled by finite element method. In detail, 88209 SPG particles and 782080 elements (FE) are employed to discretize the target, which contains a total number of 884601 nodes. All the other numerical setups including material parameters in this test are the same as those for the perforation analysis.

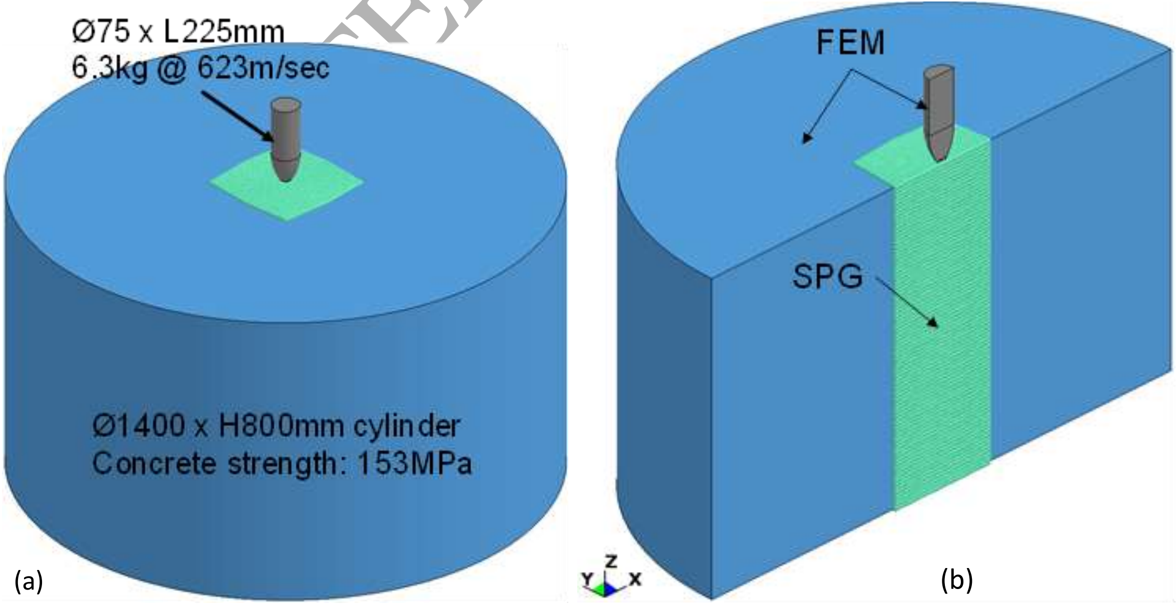

Fig. 15. Model for penetration analysis: (a) dimension, (b) schematic discretization.

Fig. 16 shows the projectile velocity and penetration depth histories in comparison with the experimental data. As shown in Fig. 16, the numerical results match the experimental results very nicely within the reliable region of test data. A relatively constant deceleration is obtained in both the numerical and the experimental results. The zero residual velocity in the numerical analysis indicates that the response is penetration, i.e., the projectile stops in the specimen. In fact, it penetrates into the target about 498 mm and then stopped. The normalized energy is shown Fig. 17. It is observed that the loss of total energy is very small and therefore, the energy conservation is satisfied.

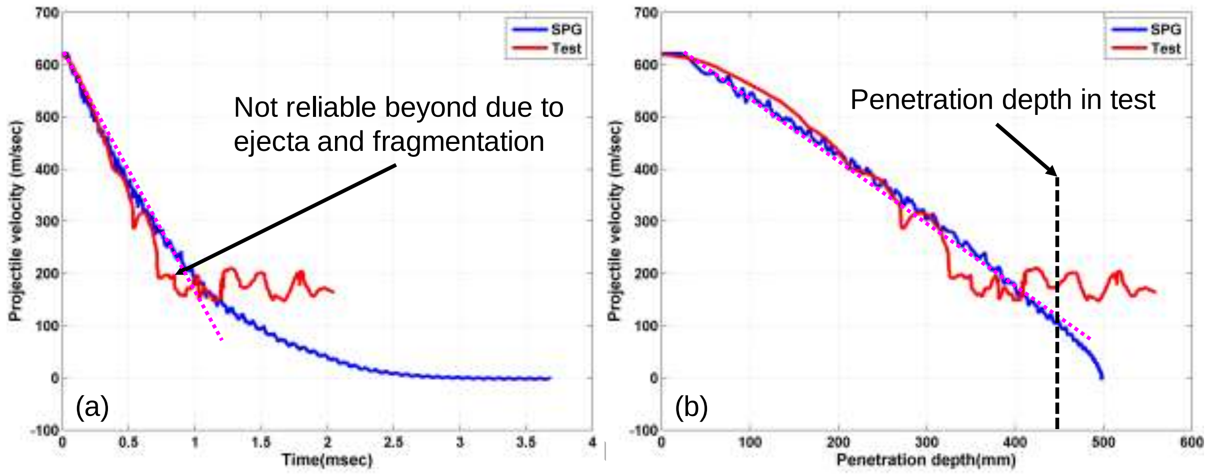

Fig. 16. Response of penetration: (a) projectile velocity time history, (b) history of projectile velocity vs penetration depth.

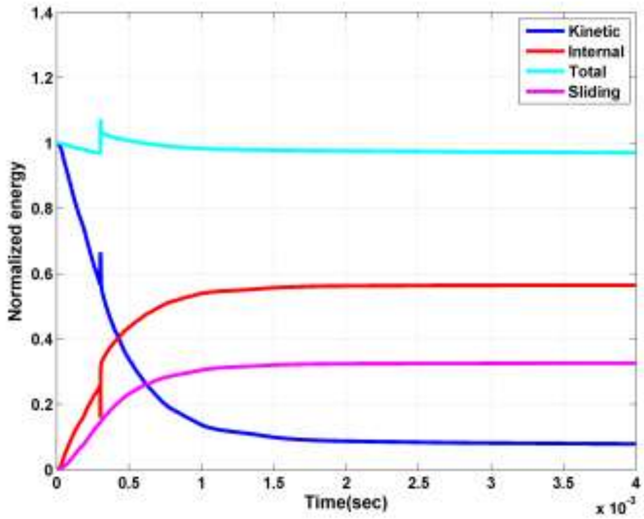

Fig. 17. Penetration responses: energy profile

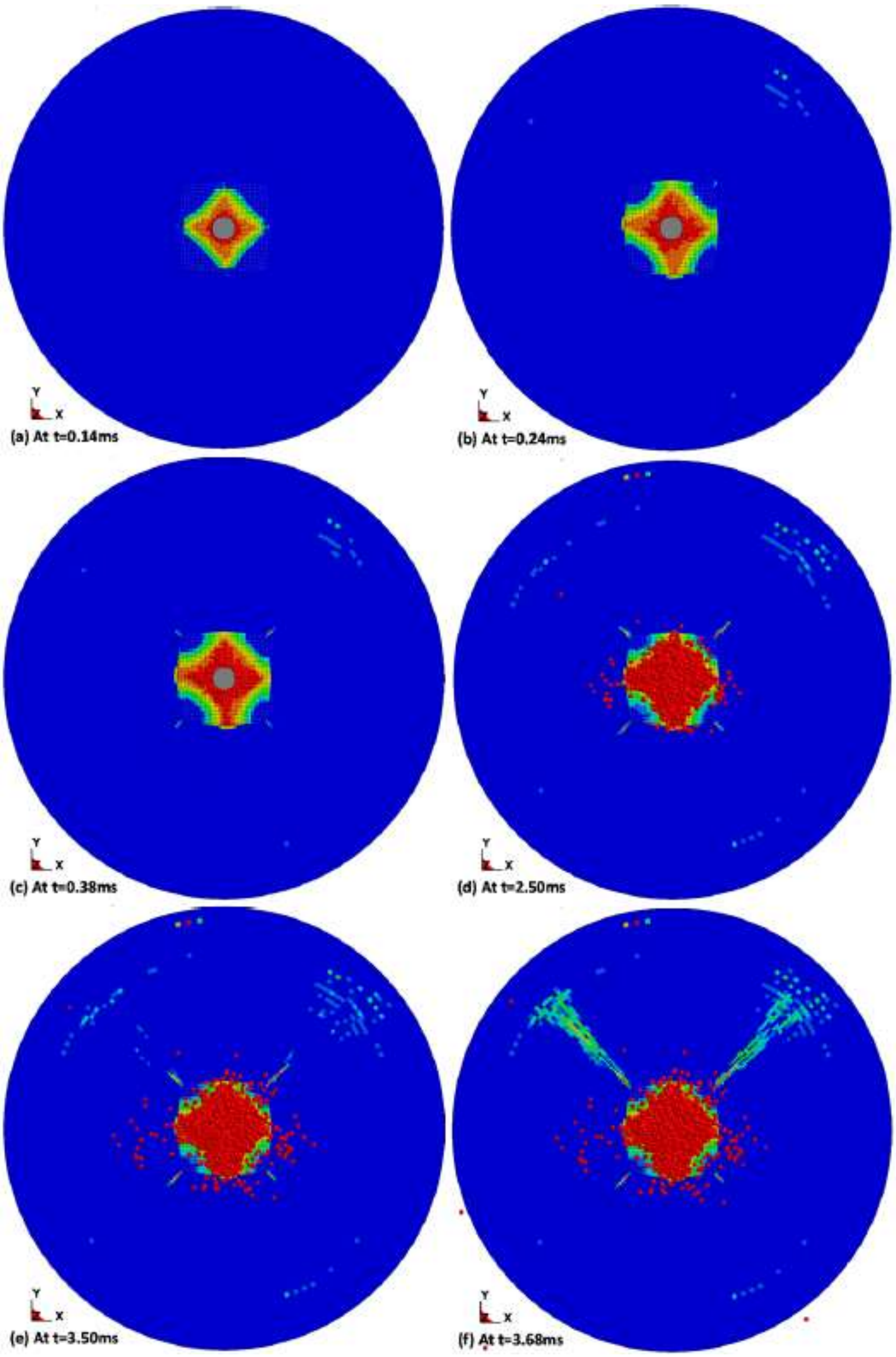

Fig. 18. Radial damage evolution in penetration response.

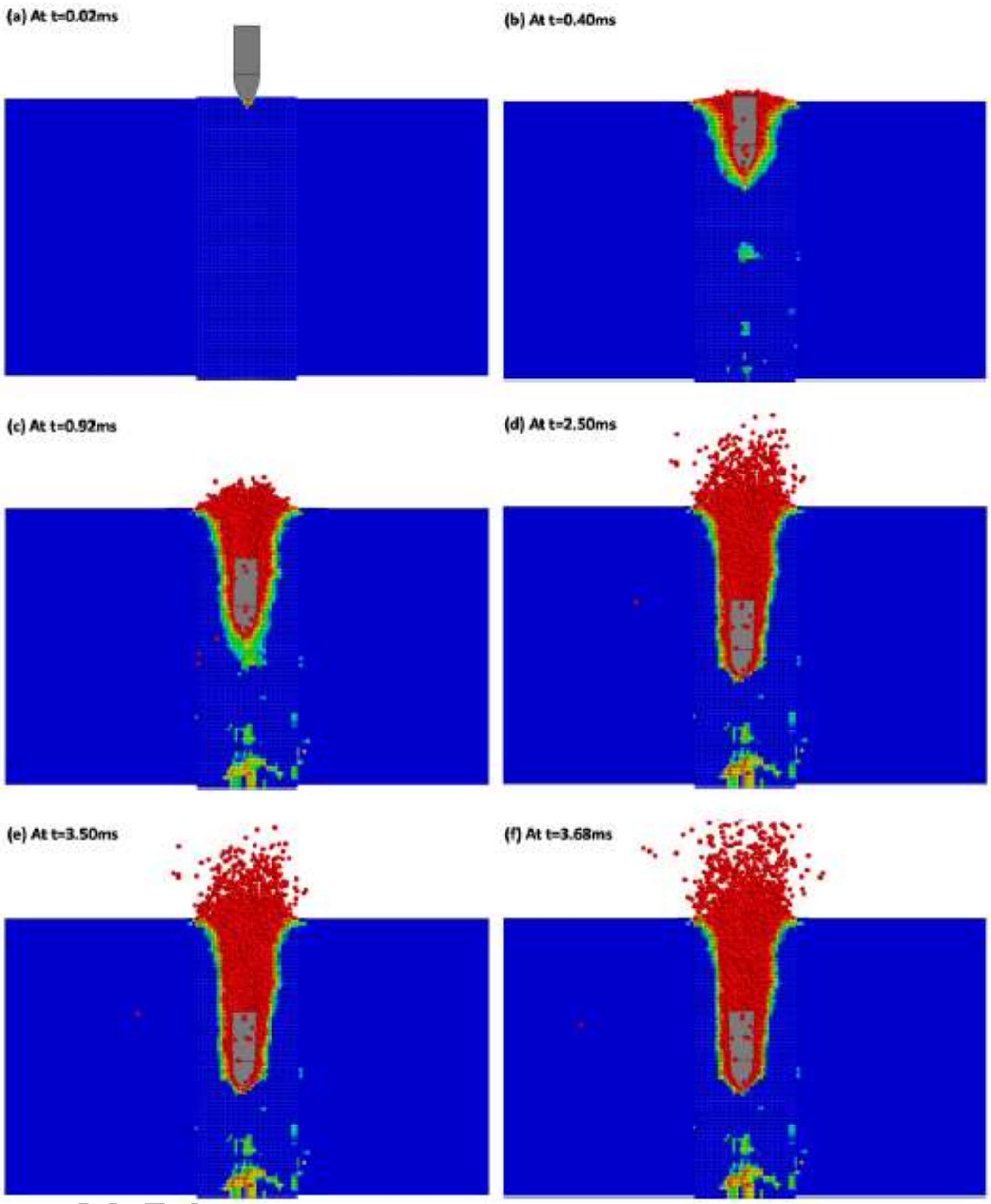

Fig. 19. Longitudinal damage evolution in penetration response.

Fig. 18 demonstrates the damage evolution in the cross – section view (on the impact face). It can be observed that damage spread out to the edge of meshfree zone at about 0.14 ms. Circumferential damage bands were noticed at around 0.24 ms, which is due to the reflected rarefaction tensile wave from the free outer surface. These damage bands were never developed enough to form macroscopic cracks until the end of simulation at 3.68 ms while the projectile has fully stopped. Radial damage bands were generated at about 0.38 ms. The radial damage bands seemed to have no significant development until 2.50 ms, and then a little evolution in the length at 3.50 ms. At the end of simulation at 3.68 ms, two long damage bands were observed. They are still called the damage bands because the damage level in these bands are not high enough to form macroscopic cracks. This is due to the fact that the specimen is strong enough to absorb all the impact energy without macroscopic fracturing.

The longitudinal damage evolution is depicted in Fig. 19 for the penetration process on the section view through the central plane of the structure. Damage starts to accumulate right away when the projectile – target contact is initiated at 0.02 ms. Tensile damage is observed from the opposite face of the impact at around 0.40 ms. This is due to the reflection of the compressive impact wave from the free bottom surface, which forms a high pressure tensile wave. Meanwhile, ejecta and debris start to be observed at this time as well. More ejecta and debris are formed as the penetration process goes further until termination of the analysis at 3.68 ms when the projectile comes to a complete stop. It is also seen that the damage profile along the impact path away from the projectile tip (i.e., bottom portion of the target) is almost finalized at 0.92 ms since there is not much difference between solutions in this region for the rest of time until termination. This is because nearly 90% kinetic energy of the projectile has been absorbed by the concrete by 0.92 ms (c.f. Fig. 16). In comparison to the result in perforation test, there is no clear plug cone in this penetration response.

6. Conclusions

The numerical challenges in modeling the penetration and fragmentation of concretes consist in dealing with high levels of deformation and material failure involving in the complex material flow due to severe shock and impact loads. In this study, we have proposed an improved meshfree methodology for the prediction of fundamental structure response and projectile characteristics in the three-dimensional concrete impact and penetration problems. The present method focuses on the generalization of our previous development in two-dimensional large deformation analysis [34] to three-dimensional formulation. Additionally, in order to extend the three-dimensional large deformation formulation to the model of the material failure in concrete impact and penetration problems, a regularized concrete damage model is implemented together with an introduction of bond-based failure criterion and self-contact algorithm in the formulation.

The computational advantages offered by the present stabilized meshfree Galerkin method are very appealing. The numerical results in this study suggest that the present method is capable of delivering a stable solution that emulates the essential concrete response and projectile characteristics. In particular, the scabbing and perforation of concrete under high velocity impact are captured in the simulation. The present method also offers a great potential for solving modern impact and penetration problems when an immersed technique [68] are built-in for the formulation and the reinforced concretes are considered in the simulation. Further developments regarding such simulation will be discussed and presented in the near future.

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